

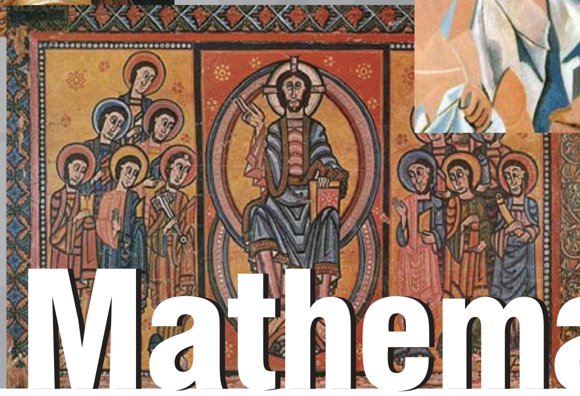
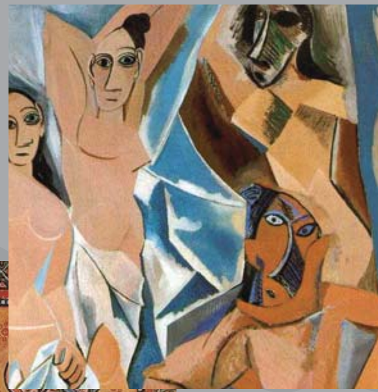

How does Mathematics

MARCIO LUIS FERREIRA NASCIMENTO AND LUIZ BARCO

HOW DOES MATHEMATICS LOOK TO YOU? YOU HAVE probably never given this a second thought and would find it a difficult question to answer.

But mathematicians often use analogies, so try this one: How does a painting look to you? What painting? Imagine any you like! Michelangelo, Rembrandt, Monet, Picasso... anyone! Or, do you prefer music? How does it look (or sound) to you? Chopin, Bach, Stravinsky, samba or jazz. Again, it's your choice!

Describing what mathematics looks like is a more difficult task than describing a painting, sculpture or music. However, we are convinced that anybody can think about mathematics as a lover of art or music thinks about his or her passions. More provocatively, we propose that when you listen to music, observe a painting, play a game, tell or read a story, you are thinking in higher-level mathematical terms. This is not a new concept; Herman Weyl explores these ideas in his brilliant book, *Symmetry*.[1]

MATHEMATICS: SIMPLICITY AND OBSTACLES

Mathematics is both a beautiful language and the simplest systematic discipline men ever created.[2] It is, for example, far simpler than physics, biology, history or economics because it concentrates on very limited aspects of reality. The simplicity of mathematical concepts almost guarantees that the facts it establishes about those concepts will also be elemental. Despite this simplicity, most people complain about the difficulty in mastering the subject and shun the study of mathematics, despite its astonishing effectiveness in almost every walk of life.

The paradox of this basically simple subject appearing difficult to learn can be explained easily. Some of the difficulties are superficial, such as the vocabulary. To denote the concepts abstracted from real objects and events, mathematicians use words that are unfamiliar to many people. Thus "triangle", "number", "set" and "equation" have specific and precise meanings when applied to the study of mathematics. However no one would contend that the acquisition of a few new words should create a major obstacle.

Another apparent and equally superficial difficulty is the use of symbols. But symbols have become commonplace in our modern, globalized world. Wherever we go, we understand international signs for "stop/go", "love", "turn right/left", "peace", "up/down", "danger", "men/women" and so on, whether our language is English, French, Japanese, German or – as are ours – Brazilian Portuguese.

A third supposed obstacle is that mathematics deals with abstractions – but then, so does art, which can be seen both figuratively and non-figuratively. Mondrian once said that "people want to see, in every work of figurative art, the desire, objectively to represent beauty, solely through form and color, in mutually balanced relations, and, at the same time, an attempt to express that which these forms, colors, and relations arouse in us."[3] Music, too, can be figurative: just listen to one of Vivaldi's *Four Seasons* to imagine birds, sunsets, happy people, and so on. In fact, abstractions should not be obstacles in mathematics because many non-mathematicians think (and feel) abstractions in their everyday life, in both figurative or non-figurative forms.

MATHEMATICS TECHNIQUE

To understand mathematics, we need to strip its underlying ideas of sophisticated and complex details, learn its objectives and uses, and understand the motivations of the men and women who created them and the genesis of its present concepts and structure. Of course, nothing can substitute for systematic study and technical mastery, but *mathematics should be distinguished from technique*. While mathematical rigor is very important, many textbooks are remarkably technical and unpleasant, especially for children. They highlight technical and relevant points, but almost never emphasize the beauty and cleverness of mathematical ideas in ways that common people can grasp easily. Like art texts, mathematical texts are rarely criticized, mainly because their technical arguments are efficiently presented even though they fail to achieve the primary educational purpose of inspiring students to appreciate mathematics (or art).

Instead of focusing on technique, we need to search for procedures that illuminate, excite and inspire – the primary objective of all education.

Look to You?

EN BREF **Comment percevez-vous les maths? Une série télévisée brésilienne sur les arts et les mathématiques, vue par plus d'un million de téléspectateurs, a remporté deux prix internationaux. Les émissions visaient principalement à expliquer que les mathématiques et les arts partagent un attrait esthétique et que tous les élèves peuvent comprendre les maths – souvent considérées la matière scolaire le plus difficile – lorsqu'elles sont présentées imaginativement, en tirant parti de ses liens étroits avec les arts. Trois exemples font le lien : entre la géométrie et la peinture classique, entre la musique et les logarithmes, ainsi qu'entre la topologie et la poésie.**

No sharp dividing line can be drawn between mathematics in its *pure* and *applied* forms; we see that it can be of *practical* importance, or it can be *beautiful* in itself. In either case, we would like to focus on the creative process, and what it involves – groping, blundering, conjecturing and hypothesizing. These activities require imagination, intuition, insight, experimentation, hard work and immense patience – important characteristics for mathematicians and artists alike.

Indeed, creative activity – or re-creative activity in the case of a student – is the heart of mathematics. Whether it is in *pure* or *applied* form does not matter: what is notable in mathematics is both its practical importance and the beauty in its arrangement of ideas. We note from history that mathematics is furthered most by men and women who are distinguished by the power of intuition rather than by the capacity to make rigorous proofs. They derive from mathematics the satisfactions that any creative activity affords. Creative potential, itself, will not produce an artist or a mathematician, but it is an important first step for these and other activities (see Example A). (For an explanation see footnote 4 on page 70.)

THE ART AND MATHEMATICS EXPERIENCE

Inspired by works like those of Freudenthal,[5] and by a desire to move the public perception of mathematics beyond the technical, government television in Brazil aired the *Arts & Mathematics* TV series in 2001. Through 13 episodes, this series attempted to show the creativity and beauty in mathematics to a wide audience of approximately one million.

The purpose of this series was first, to *entertain*, and second to *educate*, especially children and adolescents. Although learning was the secondary purpose, it is important to note that most of the mathematics presented in this series corresponded to higher level mathematical concepts at an introductory level, concepts such as topology, chaos and logic. As illustrated below, it also presented algebraic and geometric concepts. One episode (*Order in Chaos*) received two international prizes: Maeda (2001, 28th Japan Prize International Educational Program Contest) and Silver Dragon (2003, 2nd Beijing International Scientific Film Festival). Some samples from this series follow.

Example A

A few years ago, a worried mother showed one of us a division calculation done by her eight-year-old daughter. She explained that the litle girl liked to solve riddle puzzles, but became sick when given the simple task of dividing 165 by 0.5. The usual process should be to multiply both numbers by ten, proceed normally, and get 330 as the result. But the young girl showed that she had mastered the division algorithm in a *creative* way, as follows:

```
165 |0.5
```

She divided 1 by 0.5, because 2 times 0.5 is equal to one, and wrote:

```
165 |0.5
06    2
```

Next, 0.5 times twelve is equal to six, thus:

```
165 |0.5
06    2
 05  12
```

Finally, 0.5 times ten is equal to five, so:

```
165 |0.5
06    2
 05  12 +
  0  10
---------
    330
```

getting 330, the correct result.

Note that the little girl performed this operation in the usual way, with a few differences. It seems to us that for a child to make such discovery should be considered an *extraordinary* step. More, there is a remarkable comprehension of the decimal system, because:

```
165 |0.5    is translated by:
06    2     .................. 100 ÷ 0.5 = 200
 05  12     ..................  60 ÷ 0.5 = 120
  0  10     ..................   5 ÷ 0.5 =  10
---------                      ---------------
    330                                    330
```

Unfortunately, the girl's mathematics teacher considered this process wrong, and the child was disappointed. In such cases the teacher should be *prepared* to hear, *captivated* to learn, and *respectful* of a different approach. In that way, even an arid division calculation can give birth to creativity and mathematical beauty.[4]

FIGURE 1

In the Middle Age, the use of symmetry aimed to concentrate people's attention on a single focus: the rise of the soul, the sacred one. The images were represented as divine perfections. Following this sense, through its symbolism and symmetry, artists sought the attention of the observer to transmit a sensation of lightness and wellbeing. For some people this represented a harmonious meeting of men with God. A beautiful example is the *Baptism of Christ*, by Piero della Francesca (1412-1492) (see Figure 1). Although it is not possible to see identical representations in both sides of painting, all composition is organized under the symmetries of the square and the circle: the square giving forms of land, and the circle, the sky. The dove, that symbolizes the Holy Spirit, is at the center of the upper circle. The vertical line that divides the painting crosses the dove, covers the water that drains out the basin raised by Saint John the Baptist, and the united hands of Christ. Although Piero della Francesca was known as an important Renaissance painter, he wrote treatises on solid geometry and on perspective, and his works reflect those interests.

From this first example, we can imagine how an introductory geometry lesson could present mathematical concepts from art, using not only the obvious symmetries, but also the poetry and the history concerning the subject. It is important to note that the Renaissance, more than any other cultural period in world history, regarded mathematics as the *essence* of its art. Mathematicians and artists were often colleagues, swapping concepts, writing treatises and making paintings and sculptures based on each others' expertise. The example illustrated above (among others) could thus be useful as an application of geometry into art. Using music as another example, it is interesting to note that, since Johann Sebastian Bach (1685-1750), the frequencies of notes *Do – Re – Mi – Fa – Sol – La – Ti* (i.e., the chromatic scale) have been in a geometric progression (or logarithmic) of the ratio $\sqrt[12]{2}$.[6] So, when we hear a piano sonata, or jazz, or even samba, in fact we are listening to numbers! Not only music itself, but any kind of dance, with its rhythm, could be noted in mathematical terms. Aspects of counting, frequency, patterns, sequences – all these could be related in a singular way, helping to make mathematical concepts understandable to people of all ages.

The third example, which was also featured in the Brazilian TV series, draws on poetry to illustrate mathematics. According to the Brazilian poet Augusto de Campos, in 1968 Julio Plaza made his first "object" poem: two overlayed large cardboard pages projecting three-dimensional pop-ups that form through an interplay of cuttings and foldings. It occurred to de Campos to associate poetical texts to some of these objects, and so "poemobiles" were born: object-poems with words inscribed in several planes which displace themselves when the leaves are opened, allowing multiple readings.[7] The first "poemobile" was *Abre* (*Open*) (see Figure 2, p. 70). What both artists had in mind was an interdisciplinary dialogue, creative but functional, between figurative and non-figurative languages.

In the same decade, the Brazilian artist Lygia Clark (1920-1988) gave the name *Bichos* (in Brazilian Portuguese an animal, insect, or a fantastic monster) to one series of metallic leaves joined by folds that could be manipulated to form several shapes (see Figure 3, p. 70). The joints and disposals of these metallic leaves determined an enormous set of possibilities, not all easy to see at first glance. These artistic forms of view, if not strictly speaking mathematics, could be understood as *based* in a mathematical (or more precisely *topological*) form.

The practical application of examples like these is clear, but how to proceed in everyday lessons? As attributed to Euclid, "there is no royal road to geometry"; the authors consider that mathematical concepts can only be viewed from an artistic perspective by an inspirited and prepared teacher, without specific rules: there are enough rules to present algorithms and schemes in math classrooms – the main task is to *introduce better* mathematical concepts, in a free way. Thus, instead of teaching mathematics as the mere manipulation of numbers, lines and algorithms, it is both important and possible to bring the beauty of mathematics into the classroom. As the main task of any teacher is to make a subject interesting, we suggest a careful study of, among others, Lawlor, Granger, and Upitis.[8] Recent research has also suggested that successful science and mathematics learners habitually employ a set of metacognitive strategies that enable them to plan, monitor, control and regulate their own learning.[9] Researchers are also

FIGURE 2

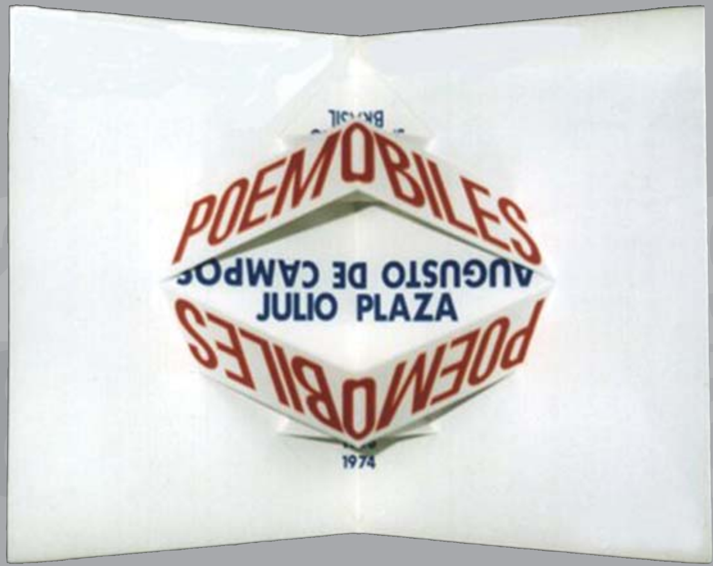

FIGURE 3

engaged in the study of undergraduate students' views of mathematics.[10] The effort to illustrate mathematical concepts using art forms could also be applied at that level.

INSTEAD OF TEACHING MATHEMATICS AS THE MERE MANIPULATION OF NUMBERS, LINES AND ALGORITHMS, IT IS BOTH IMPORTANT AND POSSIBLE TO BRING THE BEAUTY OF MATHEMATICS INTO THE CLASSROOM.

CONCLUSIONS

It should be clear to all that mastering anything, including mathematics, requires effort. However, the effort need not be unpleasant. The main problem with classroom mathematics is the method by which it is taught. Considering its nature, beauty and interest, these problems should not exist. Its vocabulary, symbolic forms and abstractions are barriers that can easily be overcome, and by distinguishing mathematical concepts from techniques, teachers should be able to reveal the beauty of the discipline. We firmly believe that the arts are an effective way to teach mathematics at an introductory level, to entertain, to demystify, and to motivate students of all ages.

Mathematicians commonly talk about beautiful theorems and beautiful proofs of theorems. Like art, the issues of beauty, simplicity, practice and pleasure are fascinating subjects; but then so is mathematics. *Quod erat demonstrandum*.

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Notes

1 H. Weyl, *Symmetry* (Princeton University Press, 1983), 5-16.
2 D. Guedj, *Numbers: The Universal Language* (UK: Harry N. Abrams, 1997), 1-20.
3 P. Mondrian, "Plastic Art & Pure Plastic Art," Circle (1937). http://www.constable.net/arthistory/glo-mondrian.html
4 Editor's note: To ensure that the algorithm illustrated is understood by Canadian readers, Sharon Friesen of the Galileo Network offers the following explanation: The child shows that she understood that what was being asked of her was how many halves are there in 165. So she initially determined how many halves there were in 1, which is 2 halves in one. Then how many halves in 6 which is 12 and lastly how many halves in 5 which is 10. All this is exactly the same as multiplying by 2.
5 H. Freudenthal, *Weeding and Sowing: Preface to a Science of Mathematical Education* (The Netherlands: Kluwer Academic Publishers, 1980), 1-24.
6 A.H. Benade, *Horns, Strings and Harmony* (USA: Dover Publications, 1992), 45-67.
7 A. de Campos and J. Plaza (*Poemobiles*, Limited Edition (Brazil: Sao Paulo, 1974), 1-2.
8 R. Lawlor, *Sacred Geometry: Philosophy and Practice* (USA, Thames & Hudson, 1989), 24-26; T, Granger, "Math is Art," *Teaching Children Mathematics* 7 (2000): 10-13; R. Upitis, "What is Arts Education Good For?" *Education Canada* 43 (2003): 24-27.
9 J. Mintzes and M-H Chiu, "Understanding and Conceptual Change in Science and Mathematics: An International Agenda Within a Constructivist Framework," *International Journal of Science and Mathematics Education* 2 (2004): 111-114.
10 A. Reid, L.N. Wood, G.H. Smith and P. Petocz, "Intention, Approach and Outcome: University Mathematics Students' Conceptions of Learning Mathematics," *International Journal of Science and Mathematics Education* 3 (2005): 567–586.